An investigation into inequalities in adult lifespan

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Foreword

This very timely report highlights how, despite huge increases in life expectancy, the gap in lifespan between richest and poorest in society is increasing for the first time since the 1870s.

Life expectancy increased in the early part of the 20th Century due to, among other things, improvements in health, clean drinking water and the introduction of vaccination.

This paper finds that in England and Wales, 5% of men that have attained the age of 30 are living on average to 96.0 years, 33.3 years longer than the lowest 10%. This gap grew by 1.7 years between 1993, when it was at its narrowest, and 2009.

ILC-UK's own 2014 research (Linking State Pension Age to Longevity), supported by Age UK, found that measures such as healthy life expectancy and disability-free life expectancy vary significantly by region and social class.

This trend is particularly worrying for society and policymakers must do more to begin to narrow this gap again. Preventing inequalities due to ill health and disability must be a priority for policy action.

But there is also a fairness challenge in the context of increasing the State Pension Age. John Cridland has been appointed by the Government as the Independent Reviewer of State Pension Age. Whilst sustainability of the state pension will be at the forefront of his concerns, Mr Cridland is also likely to consider the impact of inequalities in life expectancy.

ILC-UK believes there are likely to be significant unintended consequences of further increases to State Pension Age in 2028 if inequalities in lifespan are not addressed. Increasing State Pension Age up to levels where disability rates are higher raises concerns about transferring spending from the State Pension to disability or other working age benefits.

Increasing the State Pension Age further might also impact on the supply of carers as some people are obliged to work for longer when they would prefer to be carers.

Public policy is beginning to recognise the challenges ahead. The DWP Select Committee are conducting an Inquiry into "early drawing of the state pension". Labour have proposed a flexible state pension age so manual workers can retire earlier than other workers.

We hope that this paper adds to the evidence base to allow for an informed debate. The challenge now is how we can reduce inequalities in the future whilst, in the short term, ensure that the poorest among us do not find retirement out of their reach.

Baroness Sally Greengross
Chief Executive, International Longevity Centre - UK

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Abstract

People in the UK are living longer than ever but the gap between the oldest and shortest lived appears to be increasing. Based on data from the Human Mortality Database we measure the differences in age between the first 10% of adult deaths and the top 5% of survivors. We find that in the period from 1879 to 1939 this gap steadily closed. We argue that this reduction in inequalities in age at death was due to the benefits of clean drinking water, mass vaccination and other public health improvements which were available to everyone but which improvements were disproportionately shared by the poor relative to the rich. Although life expectancy continued to rise after 1950, the inequality gap remained roughly constant and in recent years has started to widen again – more so for men than for women. A key difference between pre-1939 and now is that deaths are increasingly from chronic rather than infectious diseases or environmental causes. Since chronic disease is often attributable to life choices such as smoking and diet, the blame for the widening must be laid increasingly at the door of individual lifestyles rather than ambient risks and hazards.

Key words: Life expectancy_ lifespan_ inequalities_ gender_ historical trend

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1. Introduction

Although life expectancy has increased enormously in recent decades, the gap in life expectancy between the shortest and longest lived is widening for the first time since the late 1870s. After a sustained period when all parts of society benefited from advances in public health, including clean drinking water, antibiotics and mass vaccination, the trend towards greater equality paused in the 1950s and, in the case of men, went into reverse in the 1990s with a re-widening of the gap.

This paper finds that 5% of men in Britain that have attained the age of 30 are living on average to 96.0 years, 33.3 years longer than the lowest 10%. This gap grew by 1.7 years between 1993, when it was at its narrowest, and 2009. The longest surviving women reach on average 98.2 years by comparison, 31.0 years more than the lowest. The female gap reached its narrowest in 2005, but has since levelled out.

We argue that unhealthy lifestyles are the main reason for these trends. Many of the big gains from public health improvements are in the past and personal choices are now much more important. Most deaths today are from age related chronic diseases and so individual life choices are more significant. According to the ONS, deaths from potentially avoidable causes account for approximately 23% of all deaths in England and Wales, with the leading causes being ischaemic heart disease in males and lung cancer in females.[2]

Men in lower socio-economic groups are the most likely to make damaging life-style choices. They put themselves in harm's way on average more than women do ….. they smoke more, drink more and there are periods in their lives when they partake in riskier activities. Since 1975 however the gap between male and female life expectancy has been reducing rapidly in the UK although not, as we shall show, the inequalities in lifespan within each gender.

Taking a longer view, figures for the past 150 years show a steadily rising life expectancy for both genders and, until 1939, a narrowing of the gap between the longest and shortest lived which we term the 'age inequality gap'. We believe the relatively abrupt switch after 1950 reflects the fact that previously achieved gains had more impact on early deaths e.g. work accidents, contagious diseases, etc. Since this time, however, changes in mortality in old age has become the key driver in determining changes in future life expectancy.

Although long run data connecting wealth and longevity is almost non-existent, the link between poverty and mortality is consistent with other research from the more recent period. Work by Smith et al using millions of UK personal medical records in the period 2005 to 2010 identifies a mortality gradient between the poorest and richest quintiles in middle age in which mortality rates in the poorest poverty quintile are up to 2.5 times higher than in the richest quintile.[3]

This finding does not mean that lack of wealth or income is directly responsible for the difference. According to Sasson (2016), who uses US data, the poorest groups are more likely to fall victim to the cumulative effects of decades of poor lifestyle and income inequality, reflecting the

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[2]http://webarchive.nationalarchives.gov.uk/20160105160709/http://www.ons.gov.uk/ons/dcp171778_362295.pdf
[3] Presentation to the Actuarial Research Conference (2014) held at the University of California, Santa Barbara, and entitled 'The Impact of Relative Poverty on Mortality Rates in the United Kingdom'. Research based on general practice patient records supplied by THIN.

repercussions of a long succession of unhealthy behaviours. In other words, it would be wrong to blame inequalities simply on contemporary phenomena such as the rise in obesity or on a historical lack of clean water, etc.

The most likely explanation is that the wealthier used to be just as vulnerable as the poor to lifestyle-related diseases. Sasson argues that the wealthier are more likely to adopt healthy behaviours and hence are better placed to avoid or defer them to later in life. Sasson further notes that healthy behaviour spreads much faster through wealthier educated networks: for example, the educated better off were the first to quit smoking. In other words, more educated people can process information relevant to their health better than less educated people.

If people do not help themselves, it is tempting for governments to intervene which partly accounts for the continuing rise of the 'nanny state'. Everybody was able to share in the benefits from water chlorination or reduced air pollution, but in today's world personal health behaviours are much more important determinants of lifespan. Tobacco consumption is a potent example of how personal choice effectively contributes to the deaths of over 100,000 people a year and shortens lives by up to ten years (e.g. see Doll et al, 2010).

With tobacco consumption heavily regulated through higher taxes, advertising and smoking bans, smoking prevalence has fallen from a high point in the 1950s of 80% of the male adult population to around 20% today. The announcement of a sugar tax on soft drinks in the March 2016 budget is a different manifestation of this approach and is a strong sign that the government has woken up to the scale of what is a significant public health challenge, which reveals itself through higher levels of obesity, type 2 diabetes and consequent health challenges.

1.2 From convergence to divergence

We term the trend prior to 1940 the 'convergent phase' – a period in which the age inequality gap between rich and poor, as measured by the dispersion in lifespan, narrowed considerably. Although life expectancy continued to rise throughout this period for both genders, the gender gap i.e. the difference between male and female life expectancy, which had been relatively small up to 1940, started to grow post-1950 reaching a peak in 1970 when female life expectancy was 5.7 years greater.

The age inequality gap throughout this period remained relatively constant for both men and women and was similar in magnitude. It is reasonable to term the post-1950 period the 'parallel phase' which corresponded to rising life expectancy but relatively little change in the relative age of death i.e. the gap in survivorship. We compared our findings against France and Italy, two other countries with similar populations and historical records, and also found a convergent phase followed by a parallel phase.

Thus far the impression gained is that these improvements are essentially a function of time. In fact observed life expectancy is subject to annual fluctuations for a variety of reasons, the most obvious examples being wars and pandemics. This was more so in the past because society has learnt to control epidemics much better and deaths from military action or environmental disasters are rarer. In the absence of these interruptions the trends in age inequalities would have been much smoother, especially in the second decade of the 20th century.

A demographic implication of our analysis is that if life expectancy had continued to improve and that reductions in lifespan had continued to narrow at similar rates to pre-1940, inequalities would have eventually disappeared resulting in a 'rectangularistion' of the survival curve (i.e. everyone lives to the same age). Because this trend fundamentally transitioned after 1950 into the 'parallel' phase, it suggests that factors operating pre-1940 had exhausted their influence.

To deal with issues raised by this brief analysis, the rest of this paper is structured as follows; the next section describes the data and methodology to compare changes through time. Section 3 examines convergence and draws upon trends in France and Italy for comparison. Section 4 shows how convergence and the compression of mortality are linked in a more general way. A concluding section discusses possible changes to society that may account for the results obtained.

2. Method of analysis

There is a burgeoning literature on demographic inequalities but the focus of attention varies enormously. Distinctions can be drawn between cross country or within country comparisons, cross sectional studies at a point or period in time, or changes through time. Similar countries, blocks or whole continents may be grouped and analysed accordingly (e.g. see Gillespie et al 2014; Edwards and Tuljapurkar 2005; or Clark 2011).

Most studies focus on the modern period since 1950 or later but some such as (Vallin and Mesle 2009) use data from the early twentieth century or much before to compare changes in life expectancy. The literature tends to be universal agreement that the degree of convergence in the globalisation era is related to economic development but it is not guaranteed or uni-directional (Jorda and Sarabia 2015).

However, there is little commonality of approach among the many studies or one single measure that captures inequality and how to define it (e.g. whether based on demography, health or income measures). The starting point for measuring demographic inequalities are usually life table measures, such as life expectancy or mortality, all of which in turn are underpinned by the registration systems in each country which record births and deaths.

The widespread availability of birth and deaths data, as compared with the relative lack of data on income and health, means that they are frequently used as a proxy for health or income inequalities (e.g. see Murray et al. 1999). Since demographic inequalities are concerned with variations in life-span, researchers seek to devise indicators to capture this variation in a single summary measure.

Life expectancy itself is a composite measure since it is an average over all lifespans and is the most familiar of all the alternatives. It is widely used, for example, to compare countries, variations within countries, or between different socio-economic or ethnic groups or between genders. This is the approach taken by Marmot (2010), for example, which found that people living in the poorest neighbourhoods in England had worse health and were likely to die sooner.

In this paper we also use life expectancy to compare the difference in longevity between men and women.   However, as useful and as familiar as life expectancy is, it suffers from the problem that it only deals with aggregate inequalities in longevity and not inequalities in lifespan. This is because it conceals variations around the average that may be suppressing important information about lifespan inequalities (e.g. within a country or region with extremes of rich and poor).

Composite measures borrowed from economics and elsewhere are also common in the inequalities literature (e.g. see Allison 1978). The Gini-coefficient, normally used to measure economic inequalities, is also used in population studies - for example, see Becker et al, (2005) or and Uddin et al (2012). Less common are measures such as Kullback-Lieber divergence which is found in d'Albis et al (2014), the Theil index in Smits and Monden (2009) or the Dispersion Measure of Mortality (Moser et al 2005).

For various reasons, aggregate measures such as these are less transparent than other available alternatives and can give false signals e.g. they may show a reduction in inequality simply because life expectancy is increasing and not because the spread in lifespan is reducing. This is obviously unhelpful if the aim is to understand the gaps in lifespan between individuals and whether they are changing for the better or worse.

Alternative methods for capturing dispersion in data include the standard deviation or variance in age at death, or the inter-quartile range (IQR) which measures the age gap between the bottom 25% and top 75% of deaths (e.g. see Wilmoth and Horiuchi 1999). Edwards (2011), for example, makes the important observation that within-country variance in lifespan is higher than between country variance, a finding which would have been difficult to ascertain by any other method.

The IQR in age of death is arguably the most intuitive of all the measures considered. In this paper we extend this idea by defining the Inter-Percentile Range or IPR as the gap in years between any two specific survival percentiles. It differs from measures such as the standard deviation and variance because it can be used to investigate the central dispersion within different ranges and so is more flexible.

Our first illustration is based on the difference between the 5th and 90th survival percentiles, though we later broaden out to include other percentiles. However, we avoid the most extreme percentiles such as 1% and 99%, as these are more prone to fluctuation due to the smaller number of deaths involved in their calculation. How the IPR and life expectancy are related is formally explained in Annex B.

Note that an IPR is an absolute measure of dispersion expressed in years. It does not follow that changes to its value are directly correlated with changes in life expectancy for the reason that an average can increase even if dispersion is unchanged or even increasing. This leads us to a key finding of this paper, namely that the relationship between dispersion (i.e. inequalities) and life expectancy changed after 1950.

The start age from which to measure inequalities is also important with different results being obtained when comparing, for example, life expectancy at birth with some other age due to variations in infant mortality, childhood mortality and so on. Hence, it would be wholly inappropriate to include these younger age groups if the focus, say, is on inequalities in life at older ages.

On this point the literature does not provide any hard-and-fast advice although it recognises that there is a difference between trends in life expectancy and/or inequalities when looking at birth compared with other ages. Examples of different start ages in the inequalities literature include 10, 15, 50 etc (e.g. see Vallin and Mesle 2009; Edwards 2011; or Mayhew and Smith 2014). In this paper we start at age 30, as we wish to focus on inequalities that account for most of the normal adult lifespan.

In effect, we are treating inequalities in infant and early adult mortality in a different category to general survivorship, without relegating its importance in any way. This is to avoid the possible confounding effects of infant mortality, but also deaths in early adult age which tends to be higher for men than women (Kannisto 2000 and 2001; Mayhew and Smith 2014).

If we look at life expectancy at age 30, we find that for much of the previous century, life expectancy generally increased year on year. Two notable exceptions are the two world wars from 1914 to 1918 and 1939 to 1945, and the unusually deadly Spanish flu epidemic between 1918 and 1920. These periods aside, we found that the IPR (using the 5th and 90th percentiles) was inversely correlated with life expectancy pre-1940 but not so post-1950.

Our data was obtained from the Human Mortality Database (HMD) which uses births-deaths data for many countries to produce complete series of deaths by calendar year and single year of age for each gender in the form of life tables. The HMD is regarded as the most reliable source on complete life tables in existence and so its use ensures greater consistency, although clearly more historical data will be less accurate than recent data.

We focus chiefly on England and Wales but, as previously noted, we also draw comparisons with France and Italy as both of these countries have long historical records available from the HMD and also have comparable sample sizes based on the annual number of deaths. Although the HMD provides complete records for each year dating back to 1841 for England and Wales, we begin our analysis in 1870 in order to accommodate Italy such that each country had the same start year.

On this basis, life expectancy data at age 30 for each country was extracted for each year from 1870 to 2010. Using the l_x values in the life tables provides information on how many lives survived to exact age x based on a radix of 100,000 individuals. We applied linear interpolation to estimate the exact ages to which 99%, 98%......1% of the population survived for both genders at annual intervals from 1870 onwards.

We used period data which means that life tables are based on current mortality rates at each age. An alternative would have been to use cohort tables to investigate changes in expected future life but two issues dissuaded us. Firstly, using cohort tables means that the last complete cohort would be for people aged 30 in 1940 (assuming a maximum age of 100), and so we would only have complete cohorts for those reaching aged 30 between 1870 to 1940. Secondly, as we later show, there are distinct historical phases which would become blurred if we had used cohort data.

3. Results

The data can be split in to different periods depending on historical focus and pattern change. In this paper, we focus our attention on the period from 1870 to 1939 and from 1950 onwards. We choose 1939 because it marks the start of the Second World War, and 1950 because it allows for a short respite after the war for the data to return to normality. We call the first period 'A' and the second period 'B'.

Tables 1 (a – d) show results for England and Wales based on life expectancy from age 30 for two 60 year periods, pre- and post-transition: these are 1879 to 1939 and from 1950 to 2010. It compares the magnitude and direction of change at six equi-spaced points in time terminating in 1939 and 2010: (a) and (b) compare age inequalities and life expectancy by gender; (c) gender differences in life expectancy; and (d) gender changes to the age inequality gap.

Table 1a shows that in the first period, male life expectancy rose from 32.2 years to 38.7 years and in the second period 40.5 years to 49.6 years, an improvement of 6.5 and 9.1 years respectively. Meanwhile, the age inequality gap between the top 5% and bottom 10% fell by 7.7 years in the first period; but in the second period it fell by only 1.2 years.

Table 1b shows that in the first period female life expectancy rose from 34.5 years to 42.1 years and in the second period from 44.8 years to 53.2 years, an improvement of 7.6 and 8.4 years. The age inequality gap between the top 5% and bottom 10% fell by 8.7 years in the first period and by 3.1 years in the second. Therefore, although the pace of reduction in inequalities slowed considerably, improvements for women were greater than for men.

The results show that as life expectancy grew, the age inequality gap decreased in the period to 1939 which we label period A. After 1950 (period B) the age inequality gap showed very little further fall and in 2010 it was almost the same as it was in 1950 in the case of males. Since the tables are based only on snapshots at six points in time, the next step is to analyse the annual changes in greater detail.

Figure 1 uses annual data from 1870 onwards. There are four series plotted in the chart: one each for male and female life expectancy at age 30 which has an identifiable trend rising from left to right; secondly, age inequalities based on the difference in years between the ages of the 5th and 90th percentiles of male and female survivors which has an identifiable downward trend from left to right until 1950.

As can be seen, in period A life expectancy made steady progress for the whole period. In period B, the increase in life expectancy initially slowed in the case of males but then accelerated after 1970. It is particularly noticeable that the difference in life expectancy between males and females grew wider in the 1970s reaching a maximum of 5.7 years in 1975. Since then there has been a steady reversal, with the gender gap in life expectancy falling to 3.7 years in 2010.

We also note that the age inequality gap changes after 1950 for both sexes. Up to that point it had been falling almost continuously for the whole of period A with the obvious exception of the period 1914 to 1920. Throughout most of period A it is noteworthy that, despite gender differences in life expectancy, the gender differences in the size of the age inequality gap were actually very small.

In period B the age inequality gap noticeably levelled out and remained relatively flat thereafter for both genders. After 1990 this situation appeared to alter with the gap for women beginning to turn downward and for men upward suggesting age inequalities were re-widening. Between 1993 and 2009, for example, the male age inequality gap increased 1.7 years to 34.2 years. This compares with a reduction in female age inequality of 0.5 years over the same period.

England and Wales

(a) Men

Year	Period A			Period B		
	1879	1909	1939	1950	1980	2010
Age reached by top 5%	84.6	85.8	87.0	88.0	89.8	95.7
Age reached by bottom 10%	39.7	43.9	49.8	53.5	56.6	62.4
Inequality gap in years	44.9	41.8	37.2	34.5	33.2	33.3
Life expectancy at age 30	32.2	35.2	38.7	40.5	42.7	49.6

(b) Women

Year	Period A			Period B		>
	1879	1909	1939	1950	1980	2010
Age reached by top 5%	86.6	88.2	90.1	91.5	94.4	98.2
Age reached by bottom 10%	40.6	46.2	52.7	57.4	61.2	67.3
Inequality gap in years	46.0	42.0	37.3	34.1	33.2	31.0
Life expectancy at age 30	34.5	37.9	42.1	44.8	48.2	53.3

(c) Difference in life expectancy

Difference in W-M life expectancy at age 30 (years)	2.3	2.7	3.3	4.3	5.5	3.7

(d) Change in gap

Gender	Change in gap 1880 to 1939	Change in gap 1950 to 2010
Men	-7.7	-1.2
Women	-8.6	-3.1

Tables 1 (a-d): Survival, age inequality gap and life expectancy at age 30 for the years 1879, 1909, 1939, 1950, 1980 and 2010 for England and Wales
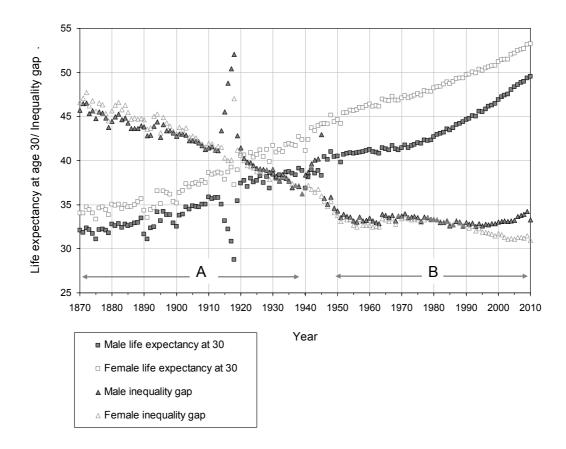
Figure 1: Chart showing the survival gap and life expectancy at age 30 for men and women in England and Wales from 1870 to 2010: (A) pre-1940; (B) post-1950)

3.1 Comparisons with France and Italy

We compared the results from England and Wales with data from France and Italy. If similar trends were observed, it would be reasonable to infer that the same demographic drivers were operating. Italy and France are chosen because of their long demographic records and similarity in population size to England and Wales. For example, we could also have used Sweden, the country with the longest historical record of all, but since it has a much smaller population the data tend to be subject to random noise and hence harder to interpret.

Tables 2 and 3 (a - d) shows the results for France and Italy in the same form as Table 1 (a - d) for England and Wales. A close examination shows that both France and Italy experienced a similar transition to England and Wales period A, i.e. a long period in which life expectancy increased and age inequalities reduced. As with England and Wales, there was also a shift in trend in period B, but with several important differences.

1. Male life expectancy in England and Wales at age 30 is currently higher than in either France or Italy, although the margin of difference post 1950 is usually about one year or less. Female life expectancy in France and Italy is currently higher than in England and Wales and has improved by greater amounts since 1950.

2. Greater differences occur when the gap between male and female life expectancies are compared. In England and Wales, the gender gap in life expectancy fell from 5.5 years to 3.7

years between 1980 and 2010 but in France it only fell from 7.3 to 6.3 years and in Italy it rose from 5.9 to 6.1 years.

3. In absolute terms the male age inequality gap is currently higher in France than in England or Wales which in turn is higher than in Italy. Currently it is 37.0 years in France as compared with 33.3 years in England and Wales and only 31.7 years in Italy. The fact that male age inequalities in Italy continued to narrow in period B by more than in period A is especially worthy of note.

4. In absolute terms, the female age inequality gap is currently lowest in Italy, standing at 28.2 years as compared with 30.6 years in France and 31.0 years in England and Wales. The level of improvement in Italy and France has been notably higher than in England and Wales. In Italy, for example, the gap closed by 5.8 years but in England and Wales by only 3.1 years.

5. If gender differences in age related inequalities are compared, we find that the gap is currently bigger in France than in either Italy or England and Wales and that it also continues to widen. In England and Wales the gender gap in age inequalities has been the lowest of all three countries and remarkably similar throughout periods A and B. However, this similarity ceased after 1990 when the gap started to re-widen.

Figures 2 and 3 provide us with an annual view of changes in demographic ageing for France and Italy which maybe compared with the earlier chart in Figure 1. All three charts show male and female life expectancy at age 30 from 1870 onwards and also male and female age inequalities between the 5th and 90th percentiles. It can be seen that the trends in each measure for men and women in England and Wales were strikingly similar whether measured in life expectancy or age inequalities especially in period A.

However, in period B whilst life expectancy continued to rise, narrowing in the age inequality gap notably slowed, suggesting that the dynamics of inequalities were changing and moving into a new phase. It is also notable that the trend in age inequalities between men and women was noticeably different in France and Italy. In France, the male trend stagnated or worsened whereas females improved considerably. In Italy the age inequalities improved for both men and women but the disparity remained wider than for England and Wales.

Despite some similarities, the trends shown in these charts and the differences, especially between the genders, are hard to account for and hence explain. In the next section we seek to show that these trends are more systematic than at first appears and that all three countries share a common pathway despite being at different points along that pathway. These points are likely to be caused by differences, not in the ambient risk factors to which both genders are exposed, but in the adoption of healthy lifestyles which, as we have argued, impact on each gender separately.

(a) Men

Year	Period A			Period B		
	1879	1909	1939	1950	1980	2010
Age reached by top 5%	84.2	84.2	85.7	88.0	90.9	96.2
Age reached by bottom 10%	40.7	41.4	44.1	50.4	53.5	59.2
Inequality gap in years	43.5	42.8	41.6	37.6	37.4	37.0
Life expectancy at age 30	33.6	33.0	34.2	39.3	42.5	49.0

(b) Women

Year	Period A			Period B		
	1879	1909	1939	1950	1980	2010
Age reached by top 5%	85.7	86.4	89.1	91.4	95.1	99.6
Age reached by bottom 10%	40.1	43.7	50.0	55.7	63.3	69.0
Inequality gap in years	45.6	42.7	39.1	35.7	31.8	30.6
Life expectancy at age 30	35.3	37.1	40.1	44.0	49.8	55.3

(c) Difference in life expectancy

	1879	1909	1939	1950	1980	2010
Difference in W-M life expectancy at age 30 (years)	1.7	4.1	5.9	4.7	7.3	6.3

(d) Change in inequality gap

Gender	Change in gap 1880 to 1939	Change in gap 1950 to 2010
Men	-1.9	-0.6
Women	-6.5	-5.2

Table 2 (a-d): Survival, age inequality gap and life expectancy at age 30 for years 1879, 1909, 1939, 1950, 1980 and 2010 for France

(a) Men

Year	Period A			Period B		
	1879	1909	1939	1950	1980	2010
Age reached by top 5%	84.4	85.1	87.3	89.5	90.4	95.6
Age reached by bottom 10%	40.6	43.6	48.3	52.5	55.8	63.9
Inequality gap in years	43.8	41.5	39.0	37.0	34.6	31.7
Life expectancy at age 30	33.1	36.6	37.7	41.0	43.2	48.5

(b) Women

Year	Period A			Period B		
	1879	1909	1939	1950	1980	2010
Age reached by top 5%	84.9	85.7	88.6	91.1	93.9	98.5
Age reached by bottom 10%	39.6	43.5	51.6	57.1	63.6	70.3
In equality gap in years	45.3	42.2	37.0	34.0	30.3	28.2
Life expectancy at age 30	33.0	37.2	40.8	43.9	49.1	54.6

(c) Difference in life expectancy

	1879	1909	1939	1950	1980	2010
Difference in W-M life expectancy at age 30 (years)	-0.1	0.6	3.1	2.9	5.9	6.1

(d) Change in inequality gap

Gender	Change in gap 1880 to 1939	Change in gap 1950 to 2010
Men	-4.8	-5.3
Women	-8.3	-5.8

Table 3 (a-c): Survival, age inequality gap and life expectancy at age 30 for years 1879, 1909, 1939, 1950, 1980 and 2010 for Italy
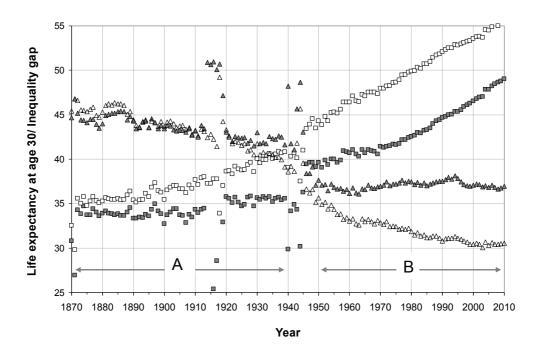
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Figure 2: Chart showing the life expectancy at age 30 and the age inequality gap for men and women in France from 1870 to 2010: (A) pre-1940 ;(B) post-1950
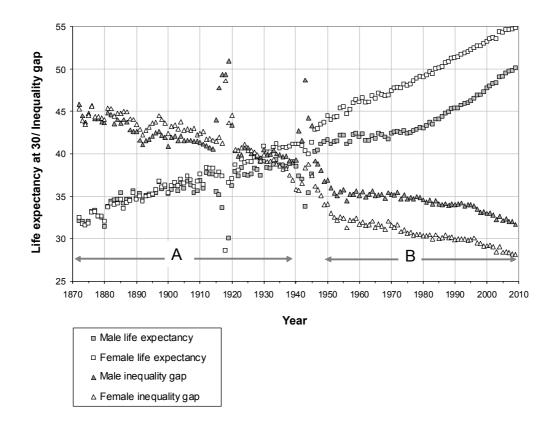

Figure 3: Chart showing the life expectancy at age 30 and the age inequality gap for men and women in Italy from 1870 to 2010: (A) pre-1940 ;(B) post-1950

4.   Relationship between lifespan inequality and life expectancy

Previously we saw that in period A as life expectancy increased the age inequality gap reduced, i.e. as people lived longer differences in lifespan became smaller. However, in period B the relationship changed such that improvements to life expectancy did not necessarily correspond to a narrowing of the gap and in addition the pace of improvement in life expectancy appeared to slow, especially in the case of men.

We also observed that improvements in life expectancy are not a monotonic process in which improvement in one year is followed by further improvement in the next. Reasons for major fluctuations in period A included major wars, pandemics, and insufficiently developed health care systems able to cope with infectious diseases or harmful bacteria. The data show that their effects were random both in their timing and shock value.
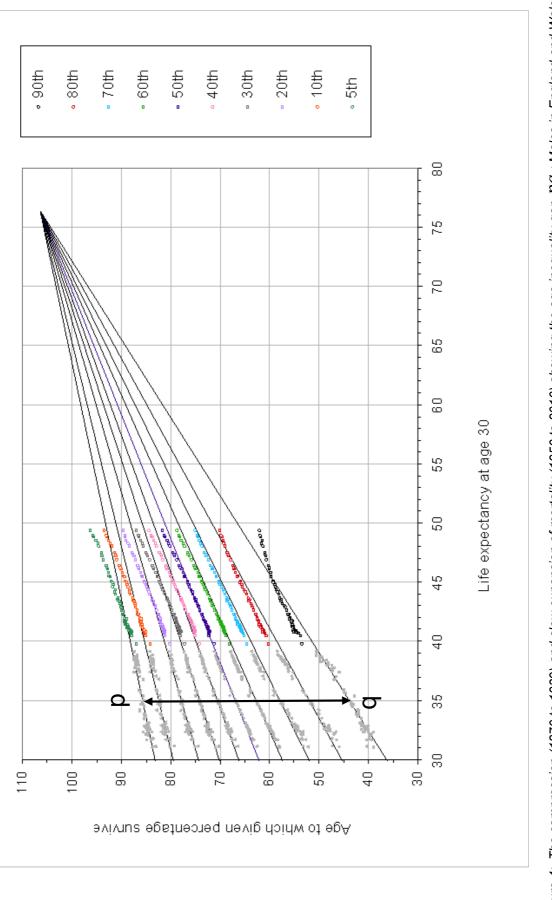
For the most part, improvements in lifespan and reduction in risks were incremental. In period A especially there were many improvements which imparted health benefits to all. These included better housing, clean water supply, improved sanitation, the discovery of antibiotics, and reductions in air pollution and poverty. In period B the data show fewer fluctuations demonstrating how society has learned to protect itself through better living conditions combined with scientific progress.
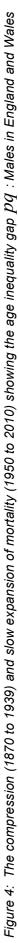
However, there will always some fluctuations in observed mortality, especially at older ages. For example, particularly cold winters can increase the death rate for pensioners. The statistical noise created by such fluctuations, especially in period A, can conceal any relationship between age inequalities and life expectancy. The questions arising therefore are: if there is a systematic relationship between life expectancy and inequalities can one be used to predict the other; secondly, has the nature of that relationship changed given the findings of the previous section?

4.1 The convergent case (period A)

Using the same data as before in order to show whether the inequality gap is increasing or decreasing, we plotted the ages of death to which different percentiles of men in England and Wales survived against life expectancy at age 30. The percentiles proceed in 10% steps starting with the age attained by 90% of survivors until the top 10% followed by a separate category for the top 5% (i.e. the 5% surviving to the highest age).

The chart shows that the highest point attained for future male life expectancy in period A is almost 40 years and in period B almost 50 years. The age inequality gap is denoted by the vertical line pq and is the difference in age between the 90th and 5th percentiles, but it is also seen to be the case that different IPRs would lead us to the same conclusion, namely that the age inequality gap for Males in England and Wales is converging.

Figure 4: The compression (1870 to 1939) and slow expansion of mortality (1950 to 2010) showing the age inequality gap *pq* : Males in England and Wales

We can also see that there is a change of pattern between the grey and coloured symbols. The age inequality gap remains broadly constant from the point at which life expectancy turned 40 (which occurred around 1950). This remained so for many years but then we observe a further change as life expectancy reaches 45 where the gap noticeably widens again suggesting that age inequalities are now starting to diverge i.e. lifespan inequalities are increasing.

As a further aid to the interpretation of Figure 4, solid lines are fitted to percentiles spanning period A and the trends then extrapolated until they meet. The results show that the age of survival was on a convergent path such that each improvement in life expectancy was accompanied by a reduction in age inequalities in lifespan. To find this point we pooled all the data for both genders in England and Wales, France and Italy so as to combine the survival experiences across all data sets from 1870 to 1939.

The method of fitting is by the method shown in Annex A. Briefly it is an iterative method in which the convergent point is varied to maximise goodness of fit. Of particular interest is the finding that there was no appreciable difference in the quality of fit in terms of the solid lines between countries and genders when all the data are pooled. We inferred that this was most probably indicative of a common evolutionary process in a survival sense, though we are not claiming it is proof.

In other words, although life expectancy in each country differs at any given point in time, in survival terms it can be argued that they are travelling along a common evolutionary path. Since this path is common to both genders and each country, it suggests that that there is no fundamental reason why male life expectancy should not eventually catch up with women as appears to be happening in England and Wales. This is assuming that there are no fundamental biological reasons to prevent this.

Our fitted percentiles also mean that had improvements in life expectancy continued on this path there would have eventually come a point, admittedly theoretical, when all inequalities would have disappeared. Diagrammatically, this would occur as pq approached zero which would be projected to occur when life expectancy at 30 had reached 76 years (i.e. everyone dies exactly at age 106). Based on statistical trends up to this point, this would not have occurred before well into the 22nd century.

The statistical parameters of the fitted solid lines are shown in Table 4 (a and b). The detailed regression results for both genders and all three countries show that:

- Percentile slope values are inversely related to survival percentiles as predicted by the stylised model described in Annex A and also show remarkable similarity, again, regardless of country and gender. This lends further support to the idea of a single convergent point.

- The slope value for the 50th percentile (i.e. median survival age) is close to a value of 1.0 showing that median survival and life expectancy track one another very closely. The proof of why they should be closely related is explained in Annex B.

- Our measure of the goodness of fit which we call $Adjusted\_R^2$ is explained further in Annex A and is based on the summation of the individual percentile values for r^2. It can be seen that the values are very similar for men and women, regardless of country.

(a) Men

Survival percentiles	England and Wales	France	Italy	Average slope value β'_p
5%	0.5013	0.5057	0.5072	0.5047
10%	0.5784	0.5792	0.5857	0.5811
20%	0.6890	0.6818	0.6878	0.6862
30%	0.7794	0.7670	0.7721	0.7728
40%	0.8645	0.8504	0.8530	0.8560
50%	0.9543	0.9414	0.9374	0.9444
60%	1.0543	1.0450	1.0353	1.0449
70%	1.1710	1.1717	1.1561	1.1663
80%	1.3165	1.3329	1.3182	1.3225
90%	1.5116	1.5410	1.5435	1.5320

Adjusted R^2	9.9204	9.1896	8.9836	9.3645

(b) Women

Survival percentiles	England and Wales	France	Italy	Average slope value β'_p
5%	0.4760	0.4884	0.5120	0.4921
10%	0.5586	0.5649	0.5921	0.5719
20%	0.6697	0.6691	0.6911	0.6766
30%	0.7620	0.7550	0.7720	0.7630
40%	0.8489	0.8368	0.8484	0.8447
50%	0.9385	0.9243	0.9269	0.9299
60%	1.0407	1.0263	1.0184	1.0285
70%	1.1661	1.1536	1.1350	1.1516
80%	1.3313	1.3358	1.3119	1.3263
90%	1.5657	1.5970	1.5748	1.5792

Adjusted R^2	9.8516	9.8126	9.6340	9.7661

Table 4: Regression slope β'_p values for different survival percentiles 1870-1939: (a) Males; (b) Females: England and Wales, France and Italy

4.2 The parallel case (period B)

We repeated this procedure for period B. A key difference is that this time we used ordinary least square (OLS) regressions to fit equations to each data point for every percentile in each population. That is to say, we ran the regressions separately for each gender and country without the constraint that the fitted percentile should pass through a convergent point. The aim now is to test the hypothesis that the inequality gap remains parallel and hence independent of changes in life expectancy.

Table 5 (a and b) show the slope values for the set of percentile regressions as for period B. A value of 1.00 would indicate that a given percentile of survivors is increasing at the same rate as life expectancy i.e. the trends are perfectly in step regardless of which percentile is compared. A value of less than one means that the increase in survival for that percentile rising more slowly than overall life expectancy, and a value greater than one more quickly.

(a) Men

Survival percentiles	England and Wales	France	Italy	Average slope value β
5%	0.892	0.871	0.791	0.851
10%	0.932	0.926	0.830	0.896
20%	0.986	1.002	0.882	0.957
30%	1.034	1.059	0.924	1.006
40%	1.072	1.103	0.969	1.048
50%	1.098	1.132	1.019	1.083
60%	1.123	1.140	1.074	1.112
70%	1.126	1.118	1.137	1.127
80%	1.075	1.031	1.193	1.100
90%	0.929	0.828	1.188	0.982

Aggregate R^2	9.963	9.956	9.937	9.952

(b) Women

Survival percentiles	England and Wales	France	Italy	Average slope value β
5%	0.852	0.777	0.796	0.808
10%	0.900	0.824	0.840	0.855
20%	0.951	0.884	0.897	0.911
30%	0.975	0.934	0.939	0.949
40%	1.001	0.982	0.974	0.986
50%	1.017	1.030	1.008	1.018
60%	1.029	1.082	1.041	1.051
70%	1.040	1.139	1.079	1.086
80%	1.076	1.194	1.127	1.132
90%	1.130	1.183	1.191	1.168

Aggregate R^2	9.943	9.983	9.981	9.969

Table 5: Regression slope β'_p values for different survival percentiles 1950-2010: (a) men; (b) women

As can be seen, the results show that slope values are very similar regardless of either gender or country, as is to be expected from a visual inspection of Figure 4. Goodness of fit is now based on R^2 i.e. the standard measure for the proportion of explained variance using OLS regression. We also introduce a summary measure of fit which we call aggregate R^2 which is simply the sum of the individual R^2s for each percentile which has a maximum possible value of 10 using our specified ten percentiles but 100 if all percentiles are used.

For both genders and each country, the table shows 99%+ of the possible maximum is achieved in all cases, indicating a very good fit to the data assuming that a linear relationship holds. Additionally, most of the slope parameter values shown in the middle columns of the tables are very close to one, as is required by the parallelism hypothesis, but there are two noteworthy points of difference:

- The post-1950 phase is not perfectly parallel. Each country in our group shows that the age to which 90% survive has a parameter of less than one for men but greater than one for women. It shows that men in the 90th percentile have not kept pace with the overall pace of change in life expectancy, unlike women.

- Between roughly the 30th and 80th percentiles, survival is running slightly ahead of the overall trend in life expectancy for both men and women (i.e. slope values are greater than one). Among other percentiles the slopes at slightly less than one meaning that survival is not quite keeping pace relative to life expectancy.

If the size of gap between the 5th and 90th percentiles is evaluated using the fitted regression lines we find that it has fallen as a percentage of life expectancy in all three countries. This is obviously better than if it had stayed the same because it would mean that relative rather than absolute inequality was rising. Also, with no sign of a convergent point as before, we can only conclude that an upper value of life expectancy has become effectively indeterminate at this point in time.

The trends in period B are measured over a 60 year period and do not take account of further indications of possible change occurring from 1990 onwards. If we concentrate our analysis on this period, we observe a further but less established change of pattern which is denoted by a widening gap in age related inequalities for males. This is obviously concerning, especially if it persists and becomes the new normality, because it shows that policies to reduce inequalities are not working as they should.

4.3 The divergent case (period C)

Since 1990, annual data for England and Wales show that the inequality gap has widened for men and fallen for women whereas previously they had been more or less on the same track as each other. If the male gap is regressed on life expectancy from 1990 to 2010 instead of from 1950 to 2010 as before, recent improvements in life expectancy are associated with an accompanying widening of the age inequality gap. For the purposes of this paper we designate the post-1990 period as period C.

Although period C is short compared with periods A and B and overlaps with B, the widening gap apparent in Figure 5 is indicative that of male age related inequalities increasing. This is in contrast to women for whom the age inequality gap is slightly narrowing. Regressions fitted to male post-1990 percentiles versus life expectancy data yield slope values that are consistent with a widening gap which we call the 'divergent phase'.

What are the reasons for this change? The mathematical reason is clear which is that men at the bottom end of the survival distribution (i.e. those with the shortest lifespan) are not keeping pace with those at the top (i.e. those with the longest lifespan). Among the highest 5% of survivors, the slope coefficient is 0.95 as compared with the 90th percentile which has a slope coefficient of only 0.69. In other words, life expectancy at the top of the distribution is increasing faster than at the bottom thereby demonstrating than inequalities in lifespan are increasing.

Therefore, whilst the longest lived are still showing very slight convergence with the rest of the population as their slope coefficient is less than one (and so they are not quite benefitting in full as the overall population's expected life increases), the main problem is that the shortest lived are being left behind. Although some of these early deaths cannot be prevented as yet e.g. accidents, genetic diseases, etc. we can be relatively certain that most of these early deaths are preventable and are lifestyle-related.

These findings are consistent with Mayhew and Smith (2014), using a different methodology, which found that the standard deviation in age at deaths was increasing for men and slightly decreasing for women. According to that research this trend looks set to continue into the next decade during by which time life expectancy between men and women at age 30 would converge (but not in terms of age related inequalities).
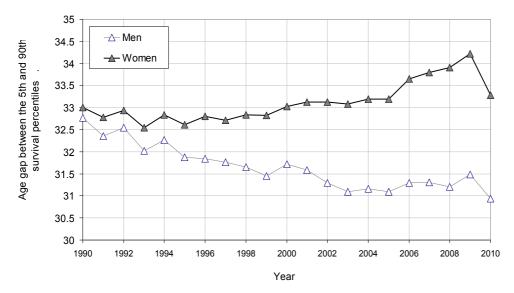

Figure 5: Change in age inequality gap between 1990 and 2010 in England and Wales during period (C)

5.   Concluding remarks

The evidence of this paper is that economic development and general improvements in life expectancy come with a narrowing of the inequalities of age at death. However, the narrowing does not continue indefinitely and there comes a point at which it slows or stops. It has been suggested that the inequality gap plateaus roughly at the point a country has completed its demographic transition from high mortality and low fertility to low mortality and low fertility (Wilmoth and Horiuchi 1999; Edwards and Tuljapurkar 2005; Wilson 2011).

We find that this pattern appears to have been replicated in England and Wales, France and Italy. In the period from 1870 to 1939, life expectancy increased and the gap in age related inequalities closed. These improvements affected men and women roughly equally, but with women having an approximate 2 to 3 year advantage in life expectancy throughout much of this period. We termed this period the convergence phase as society became more equal and the gulf in lifespan between rich and poor reduced.

After 1950, the increase in life expectancy slowed at first but then picked up again in the mid-1970s. This period was marked by a widening gap in the life expectancy of men and women in England and Wales, reaching a peak of 5.6 years in 1970. From this point, male life expectancy started to catch up with women, thereby reversing a 20-year trend. We also found that male age related inequalities ceased to narrow after 1950 and for women the improvements slowed noticeably. We termed this period the 'parallel phase' because there was little improvement in age related inequalities in spite of improvements in life expectancy.

This phase takes us to 2010 but around 1990 we observed that inequalities started to diverge with the gap among men widening relative to women. Between 1993 and 2009, for example, the male inequality gap increased by 1.7 years. As the data still cover only a relatively short period this finding may only be temporary. Nevertheless, it suggests that age related inequalities among men are currently diverging. The main reason for this is that men in the top 5% of the survival distribution are increasingly living longer and roughly equalling women in terms of longevity, but at the bottom end, age of death has levelled out.

These trends are shared by France and Italy but with some notable exceptions. As in England and Wales, life expectancy for men and women has risen but the gender gap in France and Italy is much larger. Women do better both in France and Italy in terms of life expectancy but males generally fare worse, especially French men where age related inequalities in lifespan are also much higher. In Italy, the gap in life expectancy between men and women is also higher than in England and Wales but in marked contrast to England and Wales, age related inequalities are reducing for both genders.

Various questions arise from this analysis such as what causes are driving these patterns and whether they are set to continue. In addition, if they are set to continue, is it possible to influence them in some way? In addressing this issue, it is important to separate gains in life expectancy prior to 1940 from those after 1950. Pre-1940 improvements derived from societal changes from which everybody benefited but in different measure i.e. the poorest in society closed the gap on the richest that benefitted less.

We can point to many reasons for this. Clean drinking water, improved sanitation, greater health and safety, affordable housing and cleaner air are obvious examples. It is noteworthy that the health benefits that derived from these improvements generally pre-dated the big advances in health care with the exception of mass vaccination against infectious diseases. In the modern period, these examples cannot be relied upon to produce further reductions in inequalities in the future, the implication being that the very big gains achieved are now in the past.

What has changed? There has been a transformation in the way people die. Most deaths today are from chronic rather than infectious disease or other causes including accidents. We know that nearly all chronic disease is associated with middle to old age and affects strands of society differently. The introduction referred to a study using general practice patient records which found that mortality rates in middle age to be over twice as high in the poorest quintile of households than in the richest quintile.

To address what it believed to be the root cause of the problem, the UK government introduced a target in 2003 such that by 2010 health outcomes between the best and worst areas of the country would differ by not more than 10 per cent at most, as measured by life expectancy. It had been anticipated that if more health care resources could be directed to deprived areas, the gap would narrow (House of Commons, 2009). However, we now know that the opposite occurred and that the gap in life expectancy between areas actually increased.

A difference today compared with the earlier period pre-1940 is that exposure to harm is mainly under the control of individuals rather than ambient risks which affected everyone. Tobacco smoking which causes about one in six deaths is the best example of this because it shortens lifespan by as much as ten years (see Doll et al, 2004). Smoking habits developed from the 1920s

onwards, reaching a peak in 1948 when 82% of adult males in Great Britain smoked. This compares with only 21% today who are cigarette smokers.

Much of the gain in male life expectancy in men and the closure of the gap with women who smoke less is attributable to smoking reduction (Preston and Wang 2006; Pampel 2006; Murphy and Di Cesare 2012; Mayhew and Smith 2014; Peters et al 2016). [4] However, there are many other examples of life choices that have negative effects on health e.g. excessive alcohol consumption, poor diet, and lack of exercise. All are associated with the early onset of chronic disease and also mental ill-health which is especially expensive to treat.

The increasing trend towards obesity which also reduces life expectancy is another manifestation of this tendency. Obesity is linked to poor diet and a lack of exercise, including the consumption of high levels of sugar in processed food products. In most cases, negative health outcomes of these habits are disproportionately associated with the poorest in society. All of the above suggests that extra healthcare or other financial support given to poorer areas is not as important as encouraging healthier lifestyles i.e. skewing resources to towards prevention rather than treatment.

However, it is important to stress that personal choice does not exist in a vacuum and pressures are placed on individuals through exposure to advertising, their communities and peer groups. If the poorest in society could be made healthier through greater redistribution of available heath care resources, the obesity crisis would have been solved already or even nipped in the bud but this is plainly not the case. Clearly other policy tools aimed at changing behaviour are needed to steer people towards healthy lifestyles, because if they benefit all of society benefits.

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[4] See WHO report on the global tobacco epidemic, warning about the dangers of tobacco, http://www.who.int/tobacco/global report/2011/en/

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Annex A: Estimating the point of convergence

The purpose of the annex is show how life expectancy at the convergent point can be estimated. Note that this method is appropriate for period (A) to 1939 for which convergence is applicable but not for (B) post 1950. Initially, we regressed the percentile age on life expectancy using ordinary least squares (OLS). For each survival percentile we fitted the regression:

$$y_p = \alpha_p + \beta_p e_{30} + \varepsilon_p$$

Where:

y_p is the expected age where *p* per cent of the population are still alive

e_{30} is the expected future life expectancy at age 30

α_p is the constant term for the percentile *p*

β_p is the slope for the percentile *p*

ε_p is the normally distributed random error for the percentile *p*

Although we found that OLS provides good fits to the data, the inclusion of a non-constrained constant term in the estimating equation means that the projected lines did not all cross at exactly the same value for future life expectancy due to random error.

Since the requirement for convergence is that all percentiles should pass through a single point, we imposed the condition that all regression lines should meet this requirement. An algorithm is therefore needed to ascertain what the co-ordinates of this point should be.

The easiest way to prevent this happening is to remove the constant term from the regression equation i.e. in effect, make the constant term equal to zero. This achieved by transforming the data to a temporary convergent point based at the origin (i.e. 0,0).

Let us assume that everyone will live to their 100th birthday and then die; it implies that at age 30, future life expectancy would be calculated as 70 years and the value of age of death for each percentile would be 100.

We can thus transform the data by deducting 70 from 100 the maximum age at death and then fit regression lines that to each percentile such that they all pass through the origin. In other words, if we transform our data as suggested and then for each percentile we can then fit the following regression:

$$y_p' = \beta_p' e_{30}' + \varepsilon_p'$$

Where:

y_p' is the transformed expected age where *p* per cent of the population are still alive

e_{30}' is the transformed expected future life expectancy at age 30

β_p' is the slope for the percentile *p* using the transformed data

ε_p' is the normally distributed random error for the percentile *p* using the transformed data

Figure 4 is based on fitting data to the following survival percentiles; 5th, 10th, 20th, 30th,…, 80th and 90th. Starting with a value of life expectancy of 40 years at age 30, we increased life expectancy 0.1 year steps refitting the regressions at each step.

With OLS, we can use the R-squared value as an indication of whether one model is better than another. In this case R-squared has a maximum value of one and cannot be improved upon, so it is easy to see if it is a good fit or not. However, R-squared cannot be used when it is forced to go through the origin, since R-squared is no longer constrained to a maximum value.

Instead, we can use something similar by considering the sum of the squared deviations based on each of our observations. In this way we are able to fit the best convergence point for all countries and genders separately. Since convergence implies that everyone dies at the same age, we pooled all data regardless of gender or country in order to produce a 'single fit'.

Our convergence point is defined as that giving the lowest squared deviations across all data sets. Mathematically we aimed to minimise:

$$SS_E = \sum_{pop}\sum_{p}\sum_{i=1}^{n}\varepsilon_{i,p,pop}^2 = \sum_{pop}\sum_{p}\sum_{i=1}^{n}(y_{i,p,pop} - \hat{y}_{i,p,pop})^2$$

Where:

SS_E = total sum of the squared deviations

$\varepsilon_{i,p,pop}^2$ = squared error term for the observation in the ith calendar year for the age of the pth survival percentile, for the given population

$y_{i,p,pop}$ = observation in the ith calendar year for the age of the pth survival percentile for the given population

$\hat{y}_{i,p,pop}$ = the predicted age of the pth survival percentile in the ith year given the expected life observed in the ith calendar year for the given population

We can approximate a value similar in construction and value to R^2 as an alternative. Since it is convex i.e. has a unique optimum it can be considered an alternative to R^2 for this purpose which we call $Adjusted\_R^2$:

$$Adjusted\_R_{p,pop}^2 = \frac{\sum_{i=1}^{n}(\hat{y}_{i,p,pop} - \bar{y}_{p,pop})^2}{\sum_{i=1}^{n}(\hat{y}_{i,p,pop} - \bar{y}_{p,pop})^2}$$

Where

$\bar{y}_p$ = the mean value of the pth percentile survival age for a given population

Annex B: Life expectancy, convergence and the distribution of age of death

B.1 The survival curve

Mathematically the survival curve, $S(x)$, denotes the probability of surviving to age x, whereas $e(x)$ defines life expectancy at age x. With no loss of generality, we describe a simple, stylised model from which we derive the relationship between survival, life expectancy and the distribution of ages at death.

Imagine a stationary population in which there are a constant number of births and deaths, no migration and which are subjected to the same mortality regime each year. Consider Figures B.1 (a) – (c), which show the mortality curves ABC for three such hypothetical populations *at a given point in time*. The vertical axis shows the number of survivors l_x and the horizontal axis age x.

We define the point x_1 in each case, as the onset of mortality, the age at which death begins, which can range from zero upwards. For simplicity, we assume there are no deaths before this age. We call the point where BC cuts the age axis as x_2 or the maximum age to which anyone lives.

Both x_1 and x_2 are somewhat fuzzy quantities in the real world. In developed countries, we could assume the onset of mortality (x_1) occurs from around 60 years onwards. However, our purpose is to use x_1 and x_2 as conceptually useful devices to anchor and compare distributions and mortality processes, rather than to determine them empirically.

Now imagine the age distribution of the population at another point in time. In model (a), we see that x_1 is unchanged, whilst x_2, the oldest age, has advanced to x_2' (point D). In other words, the onset of mortality is unchanged, but now some people live to older ages. The consequence of this is a decline in the mortality gradient BD compared with BC. In model (b), we see that both x_1 and x_2 have advanced by the same amount, such that the mortality gradient is the same before and after.

In model (c), x_2 has remained constant but x_1 has advanced to x_1' (point E), with the effect that the mortality gradient is steeper. For reasons that will become apparent shortly, we call (a) the divergent mortality model, (b) the parallel mortality model and (c) the convergent mortality model.

Clearly, the convergent model most closely reflects the compression hypothesis in the literature on ageing in which the shape of the survival curve becomes increasingly rectangular over time, but the other two variants do not have any comparators as far as we know.
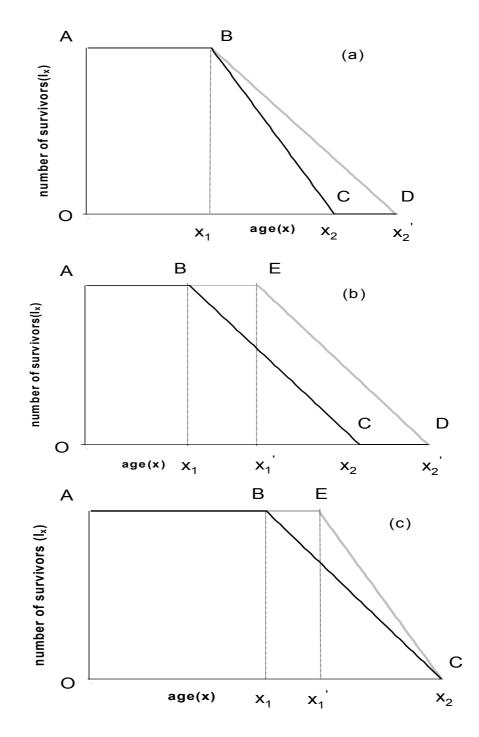
Figure B.1: Three hypothetical mortality models (a), (b) and (c)

B.2 Properties of the Simple Model

Future expectation of life

To derive the future expectation of life we need to calculate the area under the population curve divided by the starting population. In the models above we can see that all the models are formed from two basic geometric shapes – a rectangle and a right-angled triangle. If we assume the starting age is x_0 and the lives in the population at this point is l_0 then the area under the curve is:

$$l_0 \times (x_1 - x_0) + \frac{l_0}{2}(x_2 - x_1) = l_0\left(\frac{x_2 + x_1}{2} - x_0\right)$$

and hence:

$$e_{x_0} = \frac{l_0}{l_0}\left(\frac{x_2 + x_1}{2} - x_0\right) = \left(\frac{x_2 + x_1}{2} - x_0\right)$$

Which means that the expected age at death for someone currently aged x_0 is:

$$x_0 + \left(\frac{x_2 + x_1}{2} - x_0\right) = \frac{x_2 + x_1}{2}$$

Median Age of Death

The median age of death will occur when half the population has died. As we know that the survival curve in the parts where lives are observed to die is a right-angled triangle then to find the point half way down the hypotenuse we need to go half way along the base. The median age of death is therefore

$$\frac{x_2 + x_1}{2}$$

Relationship between expectation of future life and median age of death

The values derived above show that initially the expected age at death and the median age of death are the same value. As all changes in the values of x_1 and x_2 will always leave us with the same basic geometric shapes then the value of the expected age at death must always be the same as the median age of death.

Relationship of percentiles to the future expectation of life

We can calculate the theoretical relationship between cumulative mortality $p(x)$, life expectancy at x_1 and a specified age, x, that is greater than x_1. For the initial population this would be:

$$p(x) = \frac{x - x_1}{x_2 - x_1}$$

However, as the population changes we will see the values of x_1 and/or x_2 change i.e. to x_1' and/or x_2'. This will affect both the expectation of life and the ages at which a certain proportion of the population will die.

If we consider the value of $p(x)$ separately from the expectation and allowing for the possibility that $x_1' = x_1$ or $x_2' = x_2$ we can generalise to:

$$p(x) = \frac{x - x_1'}{x_2' - x_1'}$$

However, we can also calculate

$$e_{x_1} = \left(\frac{x_2' + x_1'}{2} - x_1 \right)$$

And hence we can link the percentiles with expectation of life as:

$$p(x) = \frac{x - x_1'}{2\left[e_{x_1} - (x_1' - x_1) \right]}$$

This equation is most easily understood in the context of Figure B.2 (a-c). Figure B.2a depicts the dispersion case which equates with rising inequalities or divergence. Here x_1 is assumed fixed while x_2 increases. On the vertical axis is life expectancy, which is assumed for illustrative purposes to equate with x_1. Each line represents the subsequent age to survival of 90%, 80%, 70%...... 30%, 20%, 10% and 5% of the population. In this case all percentiles converge at the origin.

In Figure B.2b, we have the same axes as before but percentiles are now parallel; that is to say, each advance in life expectancy has exactly the same proportional effect on the chances of survival at all ages above the given age and hence all age groups benefit equally. Since for this model, the increase in values for x_1 and x_2 must be the same value, each one-year increase in life expectancy advances both x_1 and x_2 by one year.

In Figure B.2c, the convergent case, all percentiles converge to a point as x_1 advances, while x_2 is held constant. Since x_2 is fixed, each one-year advance in life expectancy equates to a 1-year delay in the onset of mortality, x_1, up to a maximum of x_2, and so the rate at which the lines converge depends on advances in life expectancy. This is case which describes best the changes from 1870 to 1939 described in the paper.

Of the three models, it is readily apparent that (c) comes closest to the model originally proposed by Fries (1980), which became known as the compression hypothesis (Period A). The parallel model is observed in our data from 1950 onwards (Period B); signs of divergence (a) do not appear until 1990 (Period C)
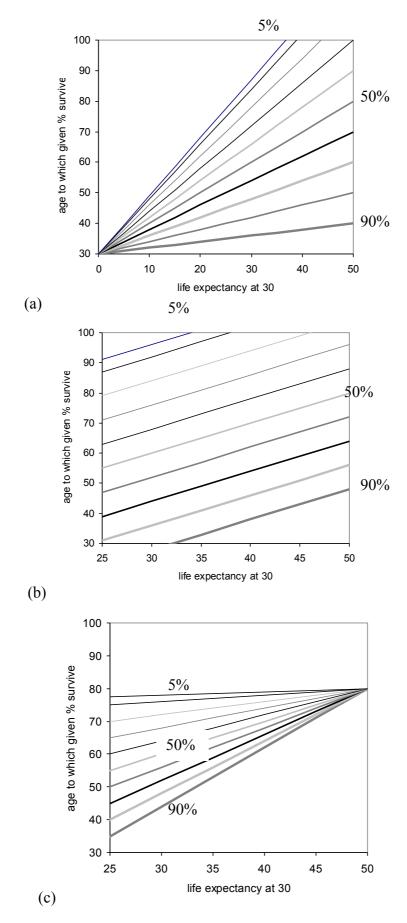
Figure B.2: Cumulative survival models: (a), (b) and (c)

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